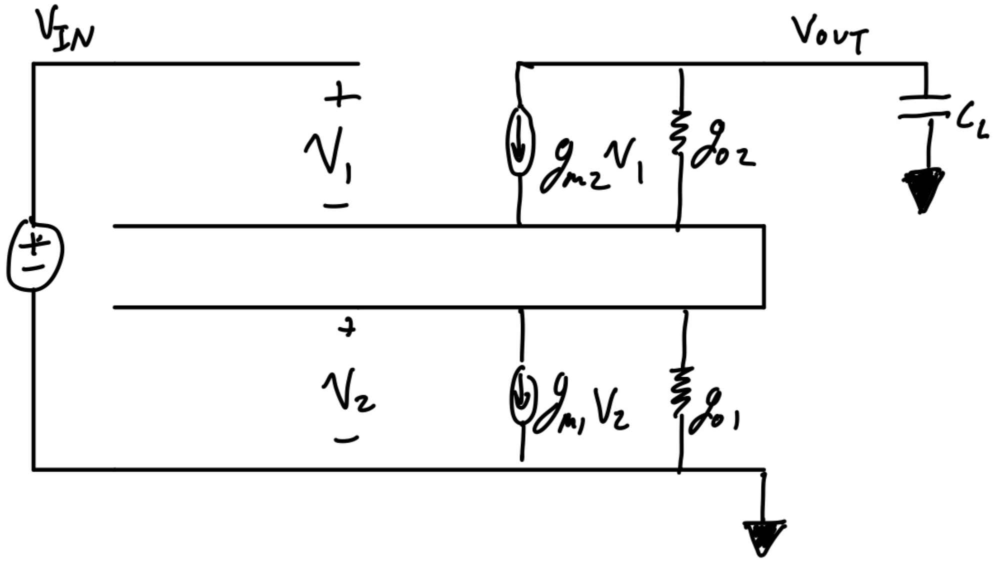
EE 435
Homework 3 Solutions
Spring 2023

Problem 1

Part A

Begin by drawing the small-signal model of the Samswrong quarter circuit.



Solving for the gain, $\frac{V_{OUT}}{V_{IN}}$, allows for g_{MEQ} and g_{OEQ} to be easily identified. This is only one strategy for finding g_{MEQ} and g_{OEQ}. Start by writing a system of equations which can be solved for V_{OUT} and V_{IN}.

$$(1)\ V_{OUT}(sC_L) + g_{m2}(V_{IN} - V_2) + (V_{OUT} - V_2)(g_{o2}) = V_{OUT}(g_{o2} + sC_L) + V_{IN}(g_{m2}) + V_2(-g_{m2} - g_{o2}) = 0$$

$$(2)\ V_2(g_{o1} + g_{m1}) + (V_2 - V_{OUT})(g_{o2}) - g_{m2}(V_{IN} - V_2) = V_{OUT}(-g_{o2}) + V_{IN}(-g_{m2}) + V_2(g_{o1} + g_{m1} + g_{o2} + g_{m2}) = 0$$

Solve Equation 1 for V_2:

$$(1.1)\ V_2 = \frac{V_{OUT}(g_{o2} + sC_L) + V_{IN}(g_{m2})}{g_{m2} + g_{o2}}$$

Substitute Equation 1.1 into Equation 2, then solve for V_{OUT} in terms of V_{IN}:

$$-g_{o2}V_{OUT} - g_{m2}V_{IN} + \frac{(g_{o1} + g_{m1} + g_{o2} + g_{m2})(V_{OUT}(g_{o2} + sC_L) + g_{m2}V_{IN})}{g_{m2} + g_{o2}} = 0$$

$$\frac{V_{OUT}}{V_{IN}} = -\frac{g_{m2}g_{o1} + g_{m2}g_{m1}}{g_{o2}g_{o1} + sC_L g_{o1} + g_{m1}g_{o2} + g_{m1}sC_L + g_{o2}sC_L + g_{m2}sC_L}$$

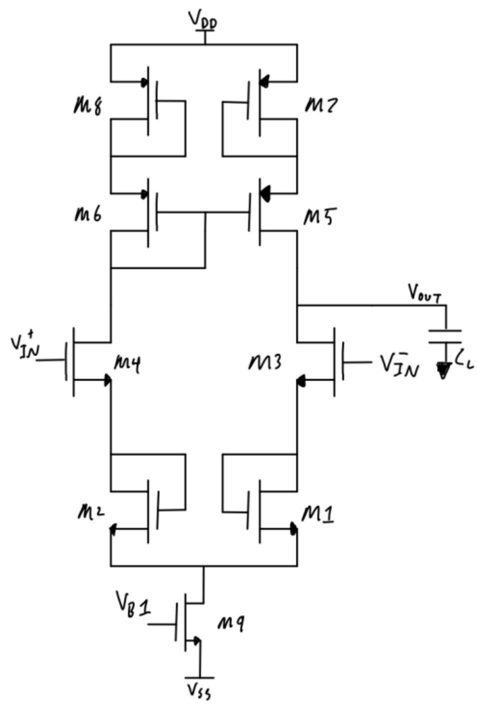
$$A_V = -\frac{\frac{g_{m2}g_{o1} + g_{m2}g_{m1}}{(g_{o1} + g_{m1} + g_{o2} + g_{m2})}}{sC_L + \frac{g_{o2}g_{o1} + g_{m1}g_{o2}}{(g_{o1} + g_{m1} + g_{o2} + g_{m2})}}$$

Recalling that $g_m \gg g_o$:

$$\frac{V_{OUT}}{V_{IN}} = A_V \approx -\frac{\frac{g_{m2}g_{m1}}{g_{m1}+g_{m2}}}{sC_L + \frac{g_{m1}g_{o2}}{g_{m1}+g_{m2}}}$$

From this, we can observe that $g_{MEQ} = \frac{g_{m1}g_{m2}}{g_{m2}+g_{m1}}$ and $g_{OEQ} = \frac{g_{m1}g_{o2}}{g_{m2}+g_{m1}}$.

Part B



By inspection, we can show that the circuit's output conductance, G_O, is $\frac{g_{m4}g_{o2}}{g_{m4}+g_{m2}} + \frac{g_{m6}g_{o8}}{g_{m6}+g_{m8}}$ and transconductance is simply $\frac{g_{m2}g_{m4}}{g_{m2}+g_{m4}}$. The amplifier's differential voltage gain and GBW are then:

$$A_V = -\frac{\frac{g_{m2}g_{m4}}{g_{m2}+g_{m4}}}{\frac{g_{m2}g_{o4}}{g_{m4}+g_{m2}} + \frac{g_{m8}g_{o6}}{g_{m6}+g_{m8}} + sC_L}$$

$$GB = \frac{\frac{g_{m2}g_{m4}}{g_{m2}+g_{m4}}}{C_L} = \frac{g_{m2}g_{m4}}{C_L(g_{m2}+g_{m4})}$$

Part C

Recall the following:

$$g_m = \frac{2I_{DQ}}{V_{EBQ}} \text{ and } g_o \approx \lambda I_{DQ}$$

Using this, we can find G_{MEQ} and G_{OEQ} in the practical parameter domain:

$$G_{MEQ} = \frac{\frac{2I_{DQ4}}{V_{EB4}} * \frac{2I_{DQ2}}{V_{EB2}}}{\frac{2I_{DQ4}}{V_{EB4}} + \frac{2I_{DQ2}}{V_{EB2}}} = \frac{\frac{4I_{DQ}^2}{V_{EB4}V_{EB2}}}{\frac{2I_{DQ}(V_{EB2}+V_{EB4})}{V_{EB4}V_{EB2}}} = \frac{2P}{(V_{DD}-V_{SS})(V_{EB2}+V_{EB4})}$$

$$G_{OEQ} = \frac{\lambda_2\lambda_4 I_{DQ}^2}{\lambda_2 I_{DQ} + \lambda_4 I_{DQ}} + \frac{\lambda_8\lambda_6 I_{DQ}^2}{\lambda_8 I_{DQ} + \lambda_6 I_{DQ}} = \frac{P}{(V_{DD}-V_{SS})}\left(\frac{\lambda_2\lambda_4}{(\lambda_2+\lambda_4)} + \frac{\lambda_8\lambda_6}{(\lambda_8+\lambda_6)}\right)$$

The DC gain and gain-bandwidth are as follows:

$$A_{DC} = \frac{G_{MEQ}}{sC_L + G_{OEQ}} = \frac{\frac{2P}{(V_{DD}-V_{SS})(V_{EB2}+V_{EB4})}}{\frac{P}{(V_{DD}-V_{SS})}\left(\frac{\lambda_2\lambda_4}{(\lambda_2+\lambda_4)} + \frac{\lambda_8\lambda_6}{(\lambda_8+\lambda_6)}\right)}$$

$$= \frac{2}{(V_{EB2}+V_{EB4})\left(\frac{\lambda_2\lambda_4}{(\lambda_2+\lambda_4)} + \frac{\lambda_8\lambda_6}{(\lambda_8+\lambda_6)}\right)}$$

$$GBW = \frac{2P}{C_L(V_{DD}-V_{SS})(V_{EB2}+V_{EB4})}$$

Problem 2

Part A

Begin by determining the gain and gain-bandwidth of the reference op amp with current mirroring. Assume λ of all devices is $0.01V^{-1}$:

$$A_0 = \frac{2}{(\lambda_1+\lambda_3)}\frac{1}{V_{EB1}} = \frac{2}{0.02} * \frac{1}{0.5} = 200$$

$$GBW = \frac{P}{(V_{DD}-V_{SS})C_L} * \frac{1}{V_{EB1}} = \frac{2P}{(V_{DD}-V_{SS})C_L} = \frac{2m}{(V_{DD}-V_{SS})C_L}$$

Now find the same two specs for the Samswrong amplifier:

$$A_0 = \frac{2}{(V_{EB2}+V_{EB4})\left(\frac{\lambda_2\lambda_4}{(\lambda_2+\lambda_4)} + \frac{\lambda_8\lambda_6}{(\lambda_8+\lambda_6)}\right)} = \frac{2}{(0.15+0.15)(0.01)} = \frac{2}{(0.15+0.15)(0.01)}$$

$$= 666.66$$

$$GBW = \frac{2P}{C_L(V_{DD}-V_{SS})(V_{EB2}+V_{EB4})} = \frac{6.666P}{(V_{DD}-V_{SS})C_L} = \frac{6.666m}{(V_{DD}-V_{SS})C_L}$$

Yes, improvements are seen in both the amplifier gain and gain-bandwidth.

Part B

The comparison of the two structures is not fair. For the comparison to be fair, they should be designed with the same V_{EB} and power budgets. Giving one structure a lower V_{EB} than another creates an unfair comparison.

Part C

To perform a fair comparison, compare the GBW and open-loop gain expressions in symbolic form, with substituting variables in. Start by comparing the gains.

$$\frac{2}{(\lambda_1+\lambda_3)}\frac{1}{V_{EB1}}\ vs.\ \frac{2}{(V_{EB2}+V_{EB4})\left(\frac{\lambda_2\lambda_4}{(\lambda_2+\lambda_4)}+\frac{\lambda_8\lambda_6}{(\lambda_8+\lambda_6)}\right)}$$

If we simplify by letting all device λs equal each other:

$$\frac{1}{\lambda}\frac{1}{V_{EB1}}\ vs.\ \frac{2}{(V_{EB2}+V_{EB4})\lambda}$$

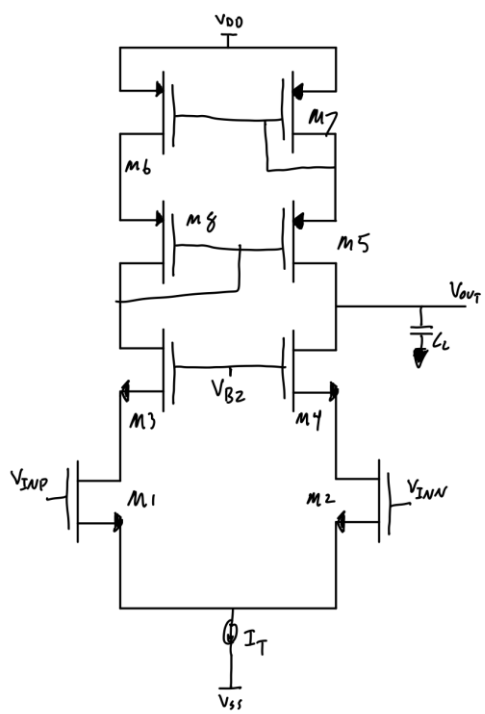
If $V_{EB2}=V_{EB1}=V_{EB4}$, the two circuits have the same gain. Now compare the GBWs:

$$\frac{P}{(V_{DD}-V_{SS})C_L}*\frac{1}{V_{EB1}}\ vs.\ \frac{2P}{C_L(V_{DD}-V_{SS})(V_{EB2}+V_{EB4})}$$

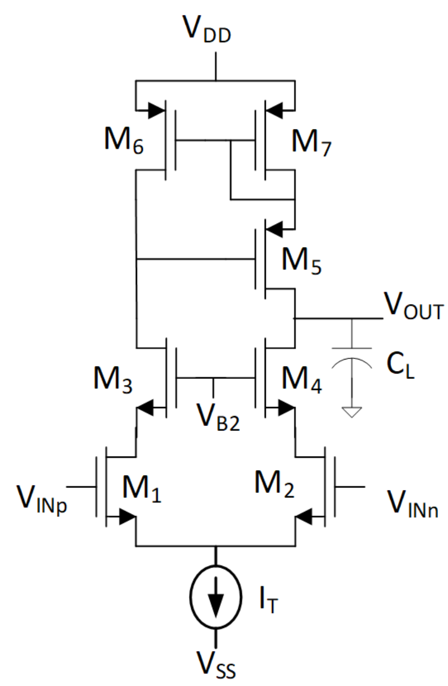
The GBW of the Samswrong circuit is less than the GBW of the reference circuit.

Problem 3

Part A



Modified Wilson Current Mirror



Wilson Mirror counterpart

Part B

To find the differential voltage gain, $A_d(s)$, start by finding G_{MEQ} for the structure. By inspection, this can be seen to be g_{m1}. Mathematically, we can see why this is true by noting that the expression for V_{OUT} in terms of V_d will inevitably end up in the following form at some point:

$$V_{OUT}(xxx)+V_d g_{m1}=0$$

Where xxx is a collection of terms. To turn this into $A_d(s)$, the $V_d g_{m1}$ term must first be subtracted from each side, and the V_d then needs to be divided. This results in g_{m1} on the right side of the expression, making it the only transconductance that matters for G_{MEQ}.

Now, find G_{OEQ}. By inspection, we can see that the output conductance of the quarter circuit is $g_{o1}\left(\frac{g_{o3}}{g_{m3}}\right)$. This is in parallel with the output conductance of the counterpart circuit. The counterpart circuit conductance is $g_{07}\left(\frac{g_{o5}}{g_{m5}}\right)$. Thus, $G_{OEQ} = g_{o1}\left(\frac{g_{o3}}{g_{m3}}\right) + g_{07}\left(\frac{g_{o5}}{g_{m5}}\right)$.

The differential gain of the amplifier is $A_d(s) = \frac{G_{MEQ}}{G_{OEQ}} = -\frac{g_{m1}}{sC_L + g_{o1}\left(\frac{g_{o3}}{g_{m3}}\right) + g_{07}\left(\frac{g_{o5}}{g_{m5}}\right)}$.

Part C

The DC gain is $= -\frac{g_{m1}}{g_{o1}\left(\frac{g_{o3}}{g_{m3}}\right) + g_{07}\left(\frac{g_{o5}}{g_{m5}}\right)}$ since this is the gain when $s = 0$. The gain-bandwidth can be found by multiplying the gain by the bandwidth. The bandwidth can be found by solving for s when the denominator is zero:

$$BW = \frac{g_{o1}\left(\frac{g_{o3}}{g_{m3}}\right) + g_{07}\left(\frac{g_{o5}}{g_{m5}}\right)}{C_L}$$

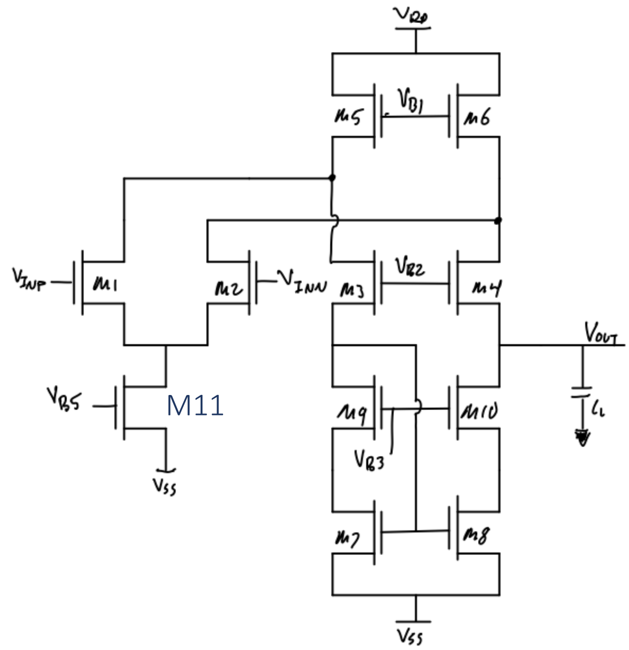
$$GBW = -\frac{g_{m1}}{C_L}$$

Part D

The gain-bandwidth is unchanged from the telescopic cascode op amp. The gain is also unchanged.

Problem 4

Part A



Part B

If $V_{DD} = 2V$ and $P = 10mW$, then the total current consumed by the amplifier must be $I_{NET} = 5mA$. One third of this current is used by the differential pair. This means we know the current and V_{EB} of all devices, so we can find the width-length of each device as follows:

$$\frac{W}{L} = \frac{2I_D}{\mu C_{ox} V_{EB}^2}$$

Assuming $\mu_n C_{ox} = 100\mu A/v^2$ and $\mu_p C_{ox} = 33\mu A/v^2$ (per the paragraph at the top of the homework assignment:

Device	**Width-Length**
M1 and M2	416.67
M3 and M4	2525
M5 and M6	3787
M7 and M8	416.6
M9 and M10	416.6
M11	833.33

Part C

Recall from lecture:

$$A_{V0} \approx \frac{2\theta}{V_{EB1}\left((\theta\lambda_1 + \lambda_5)\lambda_3 V_{EB3} + (1-\theta)\lambda_9\lambda_7 V_{EB9}\right)}$$

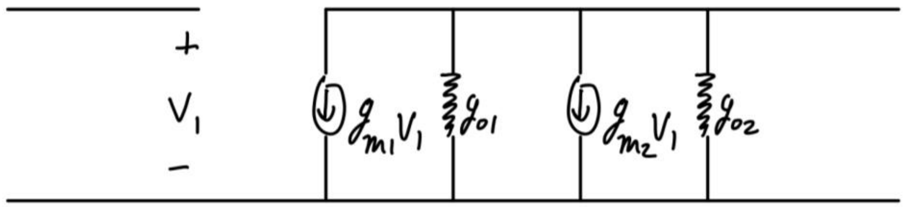
$$= \frac{2 * \frac{1}{3}}{0.2\left(\left(\frac{1}{3}\right)0.01 + 0.01\right)0.01 * 0.2 + \left(1 - \frac{1}{3}\right)0.01 * 0.01 * 0.2)} = 83333$$

$$GB = \frac{P\theta}{2V_{DD}C_L V_{EB1}} = \frac{0.00416}{C_L}$$

If $C_L = 10pF, GB = 416.67M\ rad/sec$

Problem 5

Part A



Part B

We can create a convenient practical-parameter domain design space by first expressing the current through the two-port model:

$$I_T = V_1(g_{m1} + g_{m2}) + V_{DS}(g_{o2} + g_{o1}) \approx V_1(g_{m1} + g_{m2})$$

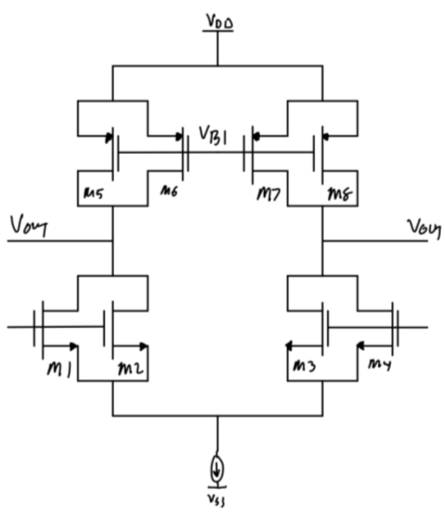
Let's express the small-signal parameters in terms of volatges and process parameters:

$$I_T \approx V_1\left(\mu C_{ox}\frac{W_1}{L_1}V_{EB1} + \mu C_{ox}\frac{W_2}{L_2}V_{EB2}\right)$$

Where $V_{EB1} = V_1 - V_{T1}$ and $V_{EB2} = V_1 - V_{T2}$.

So, a reasonable design domain is $\{V_1, \frac{W_1}{L_1}, \frac{W_2}{L_2}\}$.

Part C



Part D

Recall that gain and gain-bandwidth, in general, can be expressed as follows:

$$A_V = \frac{G_{MEQ}}{G_{OEQ}}$$

$$GBW = \frac{G_{MEQ}}{C_L}$$

We can show that $G_{MEQ} = g_{m1} + g_{m2}$ and that $G_{OEQ} = (g_{o1} + g_{o2}) + (g_{o5} + g_{o6})$. Therefore:

$$A_V = \frac{\frac{g_{m1} + g_{m2}}{2}}{g_{01} + g_{o2} + g_{o5} + g_{o6}}$$

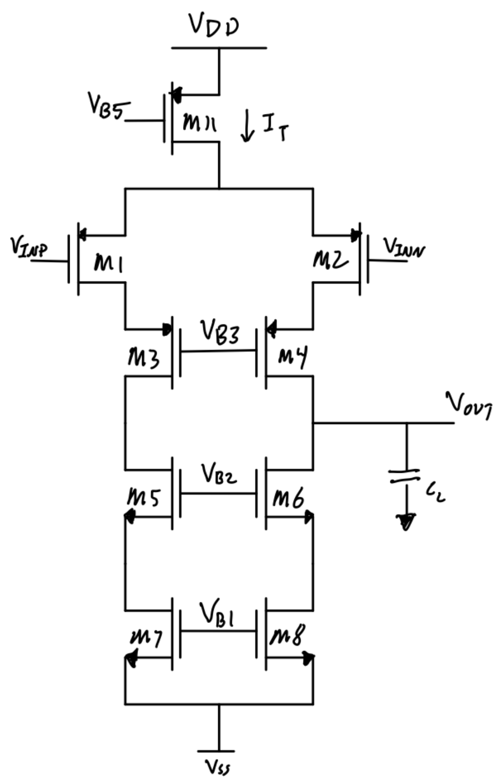
$$GBW = \frac{g_{m1} + g_{m2}}{2C_L}$$

Problem 6

Part A

If the tail current source is ideal and all bias voltages are appropriately handled with a CMFB circuit, then the amplifier will have no common-mode gain.

Part B



Part C

The maximum current that can flow through either branch is $I_T/2$. Because slew rate is simply $\frac{dV}{dt}$, we can now say that the rising and falling slew rate of the amplifier is:

$$SR^+ = SR^- = \frac{I_T}{2C_L} = 25M\frac{V}{s} = 25\ V/\mu s$$